The runaway growth of intermediate-mass black holes in dense star clusters

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ABSTRACT

We study the growth rate of stars via stellar collisions in dense star clusters. Our analytic calculations are in good agreement with direct *N*-body simulations with up to 65536 stars, performed on the GRAPE family of computers. We find that star clusters with initial half mass relaxation times $\lesssim 20$ Myr are dominated by stellar collisions, the first collisions occurring at or near the point of core collapse. The majority of collisions occur with the same star, resulting in the runaway growth of a super-massive object. This object can grow up to $\sim 0.1\%$ of the mass of the entire star cluster and could manifest itself as an intermediate-mass black hole. The phase of runaway growth lasts until mass loss by stellar evolution arrests core collapse. Star clusters older than about 4 Myr and with present-day half-mass relaxation times $\lesssim 80$ Myr are expected to contain an intermediate-mass black hole. The star cluster may sink to the Galactic center by dynamical friction before stellar evolution starts to dominate the evolution of the cluster core. In these cases the cluster dissolves close to the Galactic center and the runaway growth of the intermediate-mass black hole is terminated. Without the mediation of the star cluster the black hole cannot sink much deeper to the Galactic center and it remains in orbit where the cluster was disrupted. The mass ratio between star cluster and compact object is comparable to the ratio of the masses of Galactic bulges to their central black holes, suggesting that this process may be able to account for the super-massive black holes observed in many galaxies.

1. Introduction

Using the *Chandra* X-ray observatory, Kaaret et al. (2000; 2001) and Matsumoto et al. (2000; 2001) recently discovered nine bright X-ray sources in the irregular galaxy M82. Their brightest source (No. 7 in Table of Matsumoto et al. 2001) has a luminosity of 9×10^{40}erg s^{-1}, which corresponds to an Eddington luminosity of a $\sim 600\,\mathrm{M}_\odot$ compact object. The object appears to be variable on a time scale of about 600 seconds (Kaaret et al. 2001; Matsumoto et al. 2001) indicating that the bright X-ray source may be a single black hole with a mass of at least $600\,\mathrm{M}_\odot$.

An optical follow-up in the infrared (J, H, and K′-bands) with the CISCO instrument on the giant SUBARU telescope revealed a star cluster with an estimated mass of a few $10^6\,\mathrm{M}_\odot$ on a position consistent with the X-ray location of the intermediate-mass black hole (Harashima et al. 2001). This star cluster appears to be very young ($\lesssim 10\,\mathrm{Myr}$), as it is extremely blue and expanding shells of molecular gas have been discovered in its vicinity (Matsushita et al. 2000), typical for a star forming region. Matsushita et al. (2000) estimate that the environment has an age of only a few million years.

Many more unusually bright X-ray point sources have been discovered in the "Antennae" system (NGC 4038/4039) by Fabbiano et al. (2001), also using *Chandra*. They conclude that many of these sources may be $\gtrsim 100\,\mathrm{M}_\odot$ accreting black holes. A pair of interacting galaxies, the Antennae contain many young star clusters with characteristics similar to those found in M82 (Mengel et al. 2001). However, it is not yet clear whether the X-ray sources in the Antennae are associated with these star clusters.

In this paper we study the possibility of forming a massive compact object in a star cluster, hereby assuming that the bright X-ray sources are indeed intermediate-mass black holes, as is suggested by Matsumoto & Tsuru (2001). In fact, as explained below, it is quite natural to expect a $\sim 10^3\,\mathrm{M}_\odot$ black hole in a million $\mathrm{M}_\odot$ star cluster and there is a very distinct characteristic.

Our analysis is considerably different than the model proposed by Coleman Miller & Hamilton (2001), who argue that more than 10% of globular clusters contain a $\sim 10^3\,\mathrm{M}_\odot$ black hole. In their model the black hole grows very slowly over a Hubble time by occasional collisions with other stars. In the model explained in this paper the compact object grows much more rapidly and reaches a characteristic mass of about 0.1% of the total mass of the star cluster at birth.

2. Runaway growth of a massive object in a dense star cluster

2.1. Core collapse and the first collision

A star cluster is a self gravitating group of stars. So long as stellar evolution remains relatively unimportant, the dynamical evolution of a cluster is dominated by two-body relaxation, with characteristic time scale (Spitzer, 1987)

$$t_{\mathrm{rlx}} = \left(\frac{{R_{\mathrm{c}}}^3}{GM_{\mathrm{c}}}\right)^{1/2} \frac{N_{\mathrm{c}}}{8\ln\Lambda_{\mathrm{c}}}\,, \tag{1}$$

the half-mass relaxation time. Here G is the gravitational constant, M_{c} is the total mass of the cluster, $N_{\mathrm{c}} \equiv M_{\mathrm{c}}/\langle m\rangle$ is the number of stars and R_{c} is the characteristic (half-mass) radius of the cluster. The coulomb logarithm $\ln\Lambda_{\mathrm{c}} \simeq \ln(0.1N_{\mathrm{c}}) \sim 10$ typically. In convenient units the two-body relaxation time becomes

$$t_{\mathrm{rlx}} \simeq 1.9[\mathrm{Myr}]\left(\frac{R_{\mathrm{c}}}{[\mathrm{pc}]}\right)^{3/2}\left(\frac{M_{\mathrm{c}}}{[\mathrm{M}_\odot]}\right)^{1/2}\left(\frac{[\mathrm{M}_\odot]}{\langle m\rangle}\right)(\ln\Lambda_{\mathrm{c}})^{-1} \tag{2}$$

The dynamical evolution of the star cluster drives it into a state of core collapse (Antonov 1962; Spitzer & Hart, 1971) in which the central density runs away to a formally infinite value in a finite time. In an isolated cluster in which all stars have the same mass, core collapse occurs in a time $t_{\mathrm{cc}} \simeq 15\,t_{\mathrm{rlx}}$ (Cohn 1980).

Realistic cluster stars have a range in masses, generally from $m_{\mathrm{min}} \simeq 0.1\,\mathrm{M}_\odot$ to $m_{\mathrm{max}} \simeq 100\,\mathrm{M}_\odot$, with a mean mass $\langle m\rangle \sim 0.39\,\mathrm{M}_\odot$ (Salpeter 1955) to about $0.65\,\mathrm{M}_\odot$ (Scalo 1986). During the early evolution of the cluster, massive stars sink toward the cluster center via interactions with other stars. A star of mass

m at distance r from the cluster center drifts inward due to dynamical friction. Approximating the cluster structure as an isothermal sphere, we find (Binney & Tremaine 1987)

$$\frac{1}{r}\frac{dr}{dt} \simeq 1.2 \frac{V_c}{Gm \ln \Lambda_c} \frac{{R_c}^2}{r^{1/2}} \,. \qquad (3)$$

Here V_c is the velocity dispersion of the cluster stars. Using Eq. 2, we can integrate Eq. 3 with respect to time to obtain the dynamical friction in-spiral time scale

$$t_f = 3.3 \frac{\langle m \rangle}{m} t_{\rm rlx} \,. \qquad (4)$$

This is the time scale for a star of mass m to sink to the cluster center from a circular orbit at initial distance $r_i \gg r_{\rm core}$.

In a multi-mass system, core collapse is driven by the accumulation of the most massive stars in the cluster center. This process takes place on a dynamical friction time scale (Eq. 4). Empirically, we find, for initial mass functions of interest here, that core collapse (actually, the appearance of the first persistent dynamically formed binary systems) occurs at about

$$t_{\rm cc} =\simeq 0.20\, t_{\rm rlx} \,. \qquad (5)$$

This core collapse time is taken in the limit where stellar evolution is unimportant, i.e. where stellar mass loss is negligible and the most massive stars survive until they reach the cluster center.

The collapse of the cluster core may initiate physical collisions between stars. The product of the first collision is likely to be among the most massive stars in the system, and to be in the core. This star is therefore likely to experience subsequent collisions, resulting in a collision runaway (see Portegies Zwart et al. 1999).

The maximum mass that can be grown in a dense star cluster if all collisions happen with the same star is m_r, where

$$\frac{dm_r}{dt} = \mathcal{N}_{\rm coll} \langle \delta m \rangle_{\rm coll}. \qquad (6)$$

Here $\mathcal{N}_{\rm coll}$ and $\langle \delta m \rangle_{\rm coll}$ are the average collision rate and the average mass increase per collision (assumed independent). We now discuss these quantities in more detail.

2.2. The collision rate $\mathcal{N}_{\rm coll}$

The change in the total potential energy of a post-core-collapse star cluster during one half-mass relaxation time is on the order of 10%, largely independent of the total number of stars or the details of the cluster's internal structure (Goodman 1987). For a system without primordial binaries this energy is produced solely by heating due to dynamically formed ("three-body") binaries (Makino & Hut 1990). It is released partly in the form of scattering products which remain bound to the system, and partly in the form of potential energy removed from the system by escapers recoiling out of the cluster (Hut & Inagaki 1985). Makino & Hut argue, for an equal-mass system, that a binary generates an amount of energy on the order of $10^2 kT$ via binary–single-star scattering (where the total kinetic energy of the stellar system is $\frac{3}{2}NkT$). This quantity originates from the minimum binding energy of a binary that can eject itself following a strong encounter. Assuming that the large-scale changes in cluster potential are ultimately powered by binary heating, it follows that the required formation rate of binaries via three-body encounters is

$$n_{\rm bf} \simeq 10^{-3} \frac{N_{\rm c}}{t_{\rm rlx}} \tag{7}$$

For systems containing primordial binaries, which segregate to the cluster core, equivalent energetic arguments (Goodman & Hut 1989) lead to a similar scaling in the net rate at which binary encounters occur in the core.

The above arguments apply to star clusters comprising identical point-mass stars. In a cluster with a range of stellar masses, three-body binaries generally form from stars which are more massive than average. After repeated exchange interactions, the binary will consist of two of the most massive stars in the cluster. Conservation of linear momentum during encounters with lower mass stars means that the binary receives a smaller recoil velocity, making it less likely to be ejected from the cluster. The binary must therefore be considerably harder— $\gtrsim 10^3 kT$—before it is ejected following a encounter with another star (see Portegies Zwart & McMillan 2000).

However, taking the finite sizes of real stars into account, it is quite likely that such a hard binary experiences a collision rather than being ejected. A strong encounter between a single star and a hard binary generally results in a resonant interaction. Three stars remain in resonance until at least one of them escapes, or a collision reduces the three-body system to a stable binary. For harder binaries it becomes increasingly likely that a collision occurs instead of ejection (McMillan 1986; Pokman, Portegies Zwart & Rasio 2001, *in preparation*). In the calculations of Portegies Zwart et al. (1999) most binaries experience a collision at a binding energy of order $10^2 kT$, considerably smaller than the binding energy required for ejection. Accordingly, we retain the above estimate of the binary formation rate (Eq. 7) and conclude that the collision rate per half-mass relaxation time is

$$\mathcal{N}_{\rm coll} \simeq 10^{-3} f_{\rm c} \frac{N_{\rm c}}{t_{\rm rlx}}. \tag{8}$$

Here we introduce $f_{\rm c} \lesssim 1$, the effective fraction of dynamically formed binaries that result in a collision. Note that Eq. 8 is valid only in the limit where stellar evolution is relatively unimportant.

The most massive star in the cluster is typically a member of the interacting binary and therefore dominates the collision rate. Subsequent collisions cause the runaway to grow in mass, making it progressively less likely to escape from the cluster. The star which experiences the first collision is therefore likely to participate in subsequent collisions. The majority of collisions then involve one particular object—the runaway merger. This object is generally selected by its high initial mass and proximity to the cluster center (see Portegies Zwart et al. 1999).

2.3. The average mass increase per collision

The average mass increase per collision depends on the characteristics of the mass function in the cluster core. A lower limit for stars which participate in collisions can be derived from the degree of mass segregation in the cluster. Inverting Eq. 4 results in an estimate (still assuming an isothermal sphere) for the minimum mass of a star which can reach the cluster core in time t due to dynamical friction:

$$m_{\rm f} = 1.9 {\rm M}_\odot \left(\frac{[\rm Myr]}{t}\right) \left(\frac{R_{\rm c}}{[\rm pc]}\right)^{3/2} \left(\frac{M_{\rm c}}{[{\rm M}_\odot]}\right)^{1/2} (\ln \Lambda_{\rm c})^{-1} \;. \tag{9}$$

At time t there will be a maximum radius for each mass m inside of which stars will have segregated to the core. Dynamical friction causes the mass function in the cluster core to take on a rather curious form: the mass functions for stars with masses $m \lesssim m_{\rm f}$ and $m \gtrsim m_{\rm f}$, have the same slope as the initial mass function, but the more massive stars are overabundant because they have accumulated in the cluster center.

Once formed, the collision runaway dominates the collision cross section. The stars with which it collides are likely to be among the stars more massive than $m_{\rm f}$, simply because they are more abundant. The probability as a function of mass, however, simply follows the (amplified) initial mass function. For a Salpeter IMF, this is $dN(m) \propto m^{-2.35}dm$. Integrating this mass function from a minimum of $m_{\rm f}$ (ignoring the upper limit) results in a mean mass in the core of

$$\langle \delta m \rangle_{\rm coll} \simeq 3.9 m_{\rm f} \tag{10}$$

We ignore stars with a masses less than $m_{\rm f}$, which are dynamically unimportant (Fregeaux et al. 2001) and infrequent collision participants, due to their smaller cross sections.

Substitution of the relaxation time (Eq. 1) into Eq. 9 and Eq. 10 results in a mass increase per collision of

$$\langle \delta m \rangle_{\rm coll} \simeq 3.9 \frac{t_{\rm rlx}}{t} \langle m \rangle \ln \Lambda_{\rm c} \tag{11}$$

2.4. Lifetime of a cluster in a tidal field

The evaporation of a star cluster which fills its Jacobi surface in an external potential is driven by tidal stripping. Portegies Zwart et al (2001a) studied the evolution of young compact star clusters within ~ 200 pc of the Galactic center. Their calculations employed direct N-body integration, including the effects of both stellar and binary evolution and the external influence of the Galaxy, and made extensive use of the GRAPE-4 (Makino & Ebisuzaki 1997) special-purpose computer. The mass of a typical model cluster decreased almost linearly with time:

$$M_{\rm c} = M_i \left(1 - \frac{t}{t_{\rm disr}}\right) . \tag{12}$$

Here M_i is the mass of the cluster at birth and $t_{\rm disr}$ is the cluster's disruption time. Portegies Zwart et al (2001a) found that these clusters dissolved within 30% of the two-body relaxation time at the tidal radius (defined by substituting the tidal radius instead of the virial radius in Eq. 1). In terms of the half mass relaxation time, we find $t_{\rm disr} = 1.6$–$5.4\, t_{\rm rlx}$ depending on the initial density profile (the range corresponding to King [1966] dimensionless depths $W_0 = 3$–7). Clusters with steeper density profiles live longer.

Adopting $M_{\rm c} = N_{\rm c}\langle m \rangle$ throughout the cluster lifetime we can rewrite Eq. 12 in terms of the number of stars in the cluster. Substitution of Eqs. 8 and 11 in Eq. 6 gives

$$\begin{aligned} \frac{dm_{\rm r}}{dt} &= 4 \times 10^{-3} f_{\rm c} \frac{N_{\rm c}\langle m \rangle \ln \Lambda_{\rm c}}{t} \\ &= 4 \times 10^{-3} f_{\rm c} M_i \ln \Lambda_{\rm c} \left(\frac{1}{t} - \frac{1}{t_{\rm disr}}\right) . \end{aligned} \tag{13}$$

Integrating from $t = t_{\rm cc}$ to $t = t_{\rm disr}$ results in

$$m_{\rm r} = m_{\rm seed} + 4 \times 10^{-3} f_{\rm c} M_i \ln \Lambda_{\rm c} \left[\ln\left(\frac{t_{\rm disr}}{t_{\rm cc}}\right) + \frac{t_{\rm cc}}{t_{\rm disr}} - 1\right] . \tag{14}$$

Here $m_{\rm seed}$ is the seed mass of the star which initiates the runaway growth, most likely one of the most massive stars initially in the cluster. With $t_{\rm cc} \simeq 0.2 t_{\rm rlx}$ Eq 14 reduces to

$$m_{\rm r} = m_{\rm seed} + 4 \times 10^{-3} f_{\rm c} M_i \gamma \ln \Lambda_{\rm c}\,, \tag{15}$$

where $\gamma \simeq \ln t_{\rm disr}/t_{\rm cc} + t_{\rm cc}/t_{\rm disr} - 1 \sim 1$.

3. Results of N-body simulations

The development of the GRAPE family of special-purpose computers makes it relatively straightforward to test and tune the above simple model using direct N-body calculations. Portegies Zwart et al. (2001 *in preparation*) summarize the results of a larger series of detailed N-body calculations to study core collapse and the collision rate in dense star clusters containing up to 65536 stars. Their simulations were performed using the "Starlab" software environment (see Portegies Zwart et al. 2001b) running on the GRAPE-6 (Makino & Ebisuzaki 1997). To expand their findings, we have performed a series of simulations with 10^4 stars using the same software and hardware. These calculations were performed with initially single stars but, as just discussed, the presence of primordial binaries is not likely to change the picture qualitatively. Additional simulations of systems containing substantial numbers of primordial binaries are in progress, but are difficult, due to the complexity of following binary and multiple encounters in a large-N context. For example, a 32k simulation with 20% hard primordial binaries takes almost two orders of magnitude more computer time than a system containing no binaries.

3.1. Core collapse

In our isolated star clusters (three calculations) with 10^4 identical single point masses distributed as a Plummer model, core collapse occurs at $t_{\rm cc} \simeq 15.2 \pm 0.1\, t_{\rm rlx}$. This result is consistent with earlier calculations of e.q., Cohn (1980) and Makino (1996). Doubling the mass of 20% of the stars reduced the core collapse time to $t_{\rm cc} \simeq 7.2\, t_{\rm rlx}$. Making 20% of the stars 10 or 100 times more massive reduced the time of core collapse further to $t_{\rm cc} \simeq 1.4\, t_{\rm rlx}$ and $t_{\rm cc} \simeq 0.16\, t_{\rm rlx}$, respectively.

The more realistic models of Portegies Zwart et al. (1999) with 6144 and 12288 single stars taken from a Scalo (1986) initial mass function include mass loss from stellar evolution. The initial density distributions for these models were $W_0 = 6$ King (1966) models. Core collapse in their models occurred at $t_{\rm cc} \simeq 0.19 \pm 0.08\, t_{\rm rlx}$. The slightly later collapse compared to the models just described, containing 10^4 identical point masses and a heavy component, may be attributed to stellar mass loss, which tends to delay core collapse.

3.2. Collision rate

Relaxing the assumption of point masses to include finite stellar sizes introduces collisions into our models. In all calculations, the first collision occurred shortly after the formation of the first $\gtrsim 10$kT binary by a three-body encounter, i.e. close to the time of core collapse. When stars were given unrealistically large radii (100 times larger than normal), the first collisions occurred only slightly earlier, about 5% before deep core collapse.

The first star to experience a collision was generally one of the most massive stars in the cluster; this star then became the target for further collisions. In models with initial relaxation times greater than about 30 Myr the target star exploded in a supernova before experiencing runaway growth. The collision rates in these clusters were considerably smaller than for clusters with smaller relaxation times (see Fig. 1). This is consistent with our earlier estimates, where we argued that the onset of stellar evolution terminates the collision process. The disruption of the cluster terminates the runaway growth.

The number of collisions in the simulations of Portegies Zwart et al. (2001, *in preparation*) ranges from 0 to 24 for star clusters with up to 65536 stars (see their table 2). Fig. 1 gives the collision rate $\mathcal{N}_{\rm coll}$ per star per million years as a function of the initial relaxation time. The solid line in Fig. 1 is a fit to the simulation data, and has

$$\mathcal{N}_{\rm coll} = 2.2 \times 10^{-4} \frac{N_{\rm c}}{t_{\rm rlx}}\,, \tag{16}$$

consistent with our earlier estimate (Eq. 8) if $f_{\rm c} = 0.2$. The high quality of the fit in Fig. 1 is quite striking, especially when one bears in mind the rather large spread in initial conditions for the various models (see Portegies Zwart et al. 2002). We therefore suggest that the derived collision rate is quite universal.

Fig. 1.— Mean collision rate $f_{\rm coll} = N_{\rm coll}/N_{\rm c} t_{\rm last}$ as function of initial relaxation time for all models of Portegies Zwart et al. 2001; see their Table 2. Here $t_{\rm last}$ is the time of the last collision in the cluster. The open circles give the results of systems which are isolated from the Galactic potential (see Portegies Zwart et al 1999). Vertical bars represent Poissonian 1-σ errors. The solid line is a least squares fit to the data (see Eq. 16). The strong reduction in the collision rate for cluster with an initial relaxation time $t_{\rm rlx} \gtrsim 30$ Myr is probably real.

The increase in mass per collision in the models of Portegies Zwart et al. (1999; 2001) is consistent (within a factor of two) with the Eq. 11. Figure 2 shows the maximum mass of the runaway collision product as function of the initial mass of the star cluster. Only the left side of the figure is relevant here; we discuss the extrapolation to larger masses in Sect. 4.3. The results are consistent with the theoretical model presented in Eq. 14.

4. Discussion

Early core collapse in dense star clusters may initiate a phase of runaway stellar growth, leading to an object containing up to $\sim 0.1\,\%$ of the total cluster mass. We do not address here the state of this object, which could be a black hole or a star. If the object is a helium- or hydrogen-burning star, it may collapse into a compact object when it exhausts its central fuel. The amount of mass lost in the supernova explosion and whether the compact object receives a velocity kick are important considerations for the future evolution of the collision runaway. An extensive parameter study of the details of the supernova is beyond the scope of this paper. The basis of our analysis, however, is simple and robust to quite substantial perturbations.

We now consider the circumstances under which runaway growth may be prevented or terminated at an early stage. At the end of this section (Sect. 4.3) we briefly discuss the application of runaway growth to the formation of super-massive black holes in the bulges of galaxies.

4.1. Early termination of the collision runaway

Early termination of the runaway growth phase occurs when stellar mass loss starts to drive the expansion of the star cluster or when the star cluster is disrupted by external influences.

A star cluster in orbit around the Galactic center is subject to dynamical friction, in much the same way as dynamical friction drives the massive stars to the cluster center. This causes the cluster to spiral into the Galactic center, where it is destroyed (see Gerhard 2001). We derive here in some detail the dynamical friction time scale for a star cluster in the potential of the Galactic center.

The drag acceleration due to dynamical friction is (equation [7-18] in Binney & Tremaine, 1987)

$$a = -\frac{4\pi \ln \text{Ł}_g G^2 M_c \rho_g(R_G)}{{v_o}^2}\left[\text{erf}(X) - \frac{2X}{\sqrt{\pi}}e^{-X^2}\right] . \tag{17}$$

Here erf is the error function and $X \equiv v_o/\sqrt{2}V_G$, where V_G is the one-dimensional velocity dispersion of the stars at distance R from the Galactic center.

The mass of the Galaxy lying within the cluster's orbit at distance R ($\lesssim$ 200 pc) from the Galactic center is (Mezger et al. 1999)

$$M_G(R_G) = 4.25 \times 10^6 \left(\frac{R_G}{[\text{pc}]}\right)^{1.2} \quad \text{M}_\odot \tag{18}$$

Its derivative, the local Galactic density (see Portegies Zwart et al. 2001) is

$$\rho_G(R) \simeq 4.06 \ \times 10^5 \left(\frac{R_G}{[\text{pc}]}\right)^{-1.8} \quad \text{M}_\odot\,\text{pc}^{-3} \tag{19}$$

For in-spiral through a sequence of nearly circular orbits, the function $\text{erf}(X) - \frac{2X}{\sqrt{\pi}}\exp(-X^2)$ appearing in Eq. 17 may be determined as follows.

Following Binney & Tremaine (p. xx x), we write the equation of dynamical equilibrium for stars near the Galactic center as

$$\frac{dP}{dR_G} = -\rho_G \frac{GM_G(R_G)}{{R_G}^2} , \tag{20}$$

where $P = kT\rho/\langle m\rangle$, $\frac{3}{2}kT = \frac{1}{2}\langle m\rangle\langle v^2\rangle$. Since $\sigma^2 = \frac{1}{3}\langle v^2\rangle$, it follows that $P = \sigma^2\rho$, and Eq. 20 becomes

$$\frac{d}{dr}(\sigma^2\rho) = -\frac{\rho}{r}\,{v_o}^2 , \tag{21}$$

where v_o is the circular orbital velocity at radius R: ${V_G}^2 = GM_G(R_G)/R_G$. For $M_G \propto {R_G}^\alpha$ (see Eq. 18), and assuming that ${V_G}^2 \propto {v_o}^2 \sim {R_G}^{\alpha-1}$, we find ${V_G}^2\rho \sim {R_G}^{2\alpha-4}$, so

$$r\,\frac{d}{dr}({V_G}^2\rho) = (2\alpha - 4){V_G}^2\rho = -\rho {v_o}^2 , \tag{22}$$

and hence $X = \sqrt{2-\alpha}$. Eq. 17 then becomes

$$a = -1.2 \ln \Lambda_G \frac{GM_c}{{R_G}^2}\left[\text{erf}(X) - \frac{2X}{\sqrt{\pi}}\exp(-X^2)\right] . \tag{23}$$

For $\alpha = 1.2$, $X = 0.89$ and

$$a = -0.41 \ln \Lambda_G \frac{GM_c}{{R_G}^2} . \tag{24}$$

Again following Binney & Tremaine, defining $L = R_{\rm G} v_{\rm o}$ and setting $dL/dt = aR_{\rm G}$, we can integrate Eq. 24 with respect to time to find an inspiral time from initial radius R_i of

$$T_{\rm f} \simeq \frac{1.28}{\ln \Lambda_{\rm G}} \frac{M_{\rm G}(R_i)}{M_{\rm c}} \left[\frac{G M_{\rm G}(R_i)}{R_i^3}\right]^{-1/2} \tag{25}$$

$$\simeq 2400 \left(\frac{R_i}{1{\rm pc}}\right)^{2.1} \left(\frac{{\rm M}_\odot}{M_{\rm c}}\right) {\rm Myr} \tag{26}$$

For definiteness, we have assumed $\ln \Lambda_{\rm G} \sim 17$ ($\Lambda_{\rm G} \sim 2 \times 10^7$) in Eq. 26, corresponding to a distance of about 4 pc from the Galactic center.

4.2. Prevention of the runaway

Runaway growth in a star cluster can only occur when stellar evolution is relatively unimportant compared to the dynamical evolution of the cluster. Stellar mass loss tends to heat the cluster by loss of potential energy, and can easily reverse core collapse. This is especially true for the most massive stars, which dominate the dynamics of the cluster core and are also the first to lose substantial amounts of mass in stellar winds or supernovae. The prevention of core collapse also prevents the first collisions, and a reversal of core collapse terminates the collision runaway. As a rule of thumb, we argue that runaway growth can be prevented when the time scale for the most massive stars to segregate to the cluster center exceeds the lifetimes of those stars.

The main-sequence lifetime for stars more massive than $\sim 30\,{\rm M}_\odot$ is a rather flat function of mass: $t_{\rm ms} \simeq 21(m/{\rm M}_\odot)^{-0.42}$ Myr. For the star cluster to experience core collapse before the most massive star evolves, we require $t_{\rm cc} \simeq 0.2 t_{\rm rlx} \lesssim t_{\rm ms}(50\,{\rm M}_\odot) \sim 4$ Myr. Runaway growth can therefore be prevented in star clusters with initial relaxation times $t_{\rm rlx} \gtrsim 20$ Myr.

The maximum mass of the runaway merger for clusters which almost reach the Galactic center before being completely tidally disrupted may be calculated by substituting $t_{\rm disr} = T_{\rm f}$ in Eq. 14. (Of course, a star cluster will always be disrupted before it reaches the Galactic center.) This expression then becomes a function of

$$\frac{T_{\rm f}}{t_{\rm cc}} \simeq 6.3 \times 10^4 \left(\frac{R_{\rm i}}{\rm pc}\right)^{2.1} \left(\frac{{\rm pc}{\rm M}_\odot}{R_{\rm c} M_{\rm c}}\right)^{3/2} \tag{27}$$

We can also estimate the maximum distance to the Galactic center for which core collapse occurs before the cluster disrupts by equating $T_{\rm f} = t_{\rm cc}$. The result is $R_{\rm i} \gtrsim 0.005{\rm pc}\,(R_{\rm c} M_{\rm c}/(pc{\rm M}_\odot))^{0.71}$.

4.3. Application to massive black holes

A million solar mass star cluster formed at a distance of $\lesssim 30$ pc from the Galactic center can spiral into the Galactic center by dynamical friction before it is disrupted by the tidal field of the Galaxy (see Gerhard 2001). Only the densest star clusters survive to reach the center. These clusters are prone to runaway growth and produce massive compact objects at their centers. Upon arrival at the Galactic center, the star cluster dissolves, depositing its central black hole there. Black holes from in-spiraling star clusters may subsequently merge to form a super-massive black hole. Ebisuzaki et al. (2001) propose that such a scenario may explain the presence of the central black hole in the Milky Way galaxy.

This model explains both the bulge and the presence of a massive central black hole, and predicts that the super-massive black hole and bulge masses should be related similarly to the masses of the intermediate-mass black hole in the star cluster from which they formed. However, the ratio of stellar mass to black-hole mass is expected to be smaller for galactic bulges than for star clusters because not all star clusters produce a black hole and not all star clusters survive until the optimum black hole mass is reached. We expect, however, that the general relation between the black hole mass and that of the bulge remains valid (see Eq. 16).

Figure 2 shows the relation between the black hole mass as function of the bulge mass for Seyfert galaxies and quasars. The expression derived in Sec. 2 and the results of our N-body calculations (Sec. 3) are also included. The solid and dashed lines (Eq. 14) fit well through the N-body calculations and encloses the area of the measured black hole mass–bulge masses. On the way it passes though two other black holes mass estimates, the one in M82 and the upper limit for the globular cluster M15. [**The figure is drawn as though the SMBHs each formed from collisions in a single very massive cluster—that's that the extrapolation means. What if we assumed instead that the bulges formed from the accumulation of many million-solar-mass (say) clusters? I think the log term might go away, or at least get smaller, lowering the upper dotted line. – Steve**]

Fig. 2.— The mass after a period of runaway growth as a function of the mass of the star cluster. The solid line gives $m_{\rm r} = 30 + 8 \times 10^{-4} Mc \ln M_{\rm c}/{\rm M}_\odot$ (see Eq. 6). For cluster with $M_{\rm c} \gtrsim 10^7\,{\rm M}_\odot$ this line is extended as a dashed line. The bottom dashed line shows $0.01 m_{\rm r}$. The five error bars to the left give the results of the direct N-body calculations from Portegies Zwart et al. (2001). The data are averages of the models presented in their Table 2; from left to right: 4k stars (models KML101, KML111 and KML112), 6k (model 6k6X10), 12k (models RxW4 and 12k6X10), 14k (model KML144) and 64k (model N64R6r36). The down pointed arrow gives the upper limit for the mass of a compact object in the globular cluster M15 (van der Marel, 2001) and the error bar to the right gives the mass estimate for the compact object associated with Chandra source #7 in the irregular Galaxy M82 (Matsumoto & Tsuru 1999). The Milky Way is presented by the asterisk using the bulge mass from Dwek (et al. 1995) and the black hole mass from Eckart & Genzel (1997) and Ghez (2000). Bullets and triangles (upper right) represent the bulge masses and measured black hole mass of Seyfert galaxies and Quasars, respectively (both from Wandel 1999; 2001). The dotted lines gives the range in solutions to a least squares fits to the bullets and triangles (Wandel 2001).

5. Conclusions

We study the runaway growth of a single star in a dense star cluster using a combination of complementary approaches. Our semi-analytic analysis is supported by detailed N-body calculations in which the effects of stellar evolution, stellar dynamics, binary evolution and the perturbing effect of a background potential are self consistently taken into account.

Star clusters with an initial relaxation time $t_{\rm rlx} \lesssim 20$ Myr experience a phase of runaway growth. In this phase a single seed star grows to a mass of about 0.1% of the total mass of the star cluster.

The first collision occurs at the moment the cluster core collapses. This happens at about $0.2\,t_{\rm rlx}$ but no later than about 4 Myr. The star which experiences the first collision becomes the target for further collisions, initiating runaway growth. The growth phase is terminated by (1) the disruption of the cluster in the tidal field of the Galaxy (at $t \lesssim 5t_{\rm rlx}$) or (2) the reversal of core collapse by mass loss from the evolving stellar population (after about 20 Myr).

A star cluster can survive for longer than $5\,t_{\rm rlx}$ if, for example, it did not initially fill its Jacobi surface ("Roche lobe") in the Galactic tidal field. (Examples are NGC 3603 and R 136, the dense star cluster in the 30 Doradus region in the Large Magellanic cloud.) Such clusters go though a phase of runaway stellar growth, but recover after stellar mass loss drives the re-expansion of the cluster core.

From an observational point of view, a tidally limited cluster experiences three very distinct evolutionary phases: a pre-collapse phase until $0.2\,t_{\rm rlx}$, a phase of deep core collapse (from $0.2\,t_{\rm rlx}$ to about 20 Myr), followed by an expansion phase leading to the disruption of the cluster. During core collapse the cluster half-mass radius expands, causing the mean relaxation time to increase by a factor of 4 (see Portegies Zwart et al. 2001). A cluster in the last phase will be observable with a current relaxation time smaller than about $4 \times 20\,{\rm Myr} = 80\,{\rm Myr}$. The clearest indication of its previous phase of core collapse and runaway growth is the presence of a central compact object with a mass $\lesssim 0.1\%$ of the initial cluster mass. The cluster may also be relatively depleted in low-mass compact objects (stellar mass black holes and neutron stars), as these are consumed during the runaway growth phase.

Star clusters with $t_{\rm rlx} \gtrsim 20\,{\rm Myr}$ do not experience a phase of runaway growth, as core collapse is prevented by mass loss from the most massive stars. These clusters may experience core collapse after $\sim 100\,{\rm Myr}$, when stellar evolution slows (Takahashi & Portegies Zwart 1999). This later core collapse, however, does not lead to a phase of runaway growth.

As a bonus, our simple model explains in general terms the relation between the mass of a galactic bulge and that of its central black hole.

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